Updating the Hamiltonian Problem - A Survey

by

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ABSTRACT:
This is intended as a survey article covering recent developments in the area of hamiltonian graphs, that is, graphs containing a spanning cycle. This article also contains some material on related topics such as traceable, hamiltonian-connected and pancyclic graphs and digraphs, as well as an extensive bibliography of papers in the area.

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Section 0. Introduction

The hamiltonian problem; determining when a graph contains a spanning cycle, has long been fundamental in Graph Theory. Named for Sir William Rowan Hamilton, this problem traces its origins to the 1850's. Today, however, the flood of papers dealing with this subject and its many related problems is at its greatest; supplying us with new results as well as many new problems involving cycles and paths in graphs.

To many, including myself, any path or cycle question is really a part of this general area. Although it is difficult to separate many of these ideas, for the purpose of this article, I will concentrate my efforts on results and problems dealing with spanning cycles (the classic hamiltonian problem) or related topics that are usually stronger in nature (pancyclic, hamiltonian - connected, etc.). I shall not attempt to survey the weighted version, the traveling salesman problem, or any of its related questions. For material on this problem see [196]. I shall further restrict my attention primarily to work done since the late 70's, however, for completness, I shall include some earlier work in several places. For an excellent general introduction to the hamiltonian problem, the reader should see the article by J. C. Bermond [37]. Those not familiar with this topic or with graphs in general are advised to begin there. Further background and related material can be found in the following related survey articles: [49], [41], [197],[318],[88], [28] and [220].

This article concludes with a rather extensive list of references; far more than could be discussed within this paper. I have also tried to include the Math Reviews reference whenever possible. I hope this will be of use to those interested in research problems in this field.

Throughout this article we will consider finite graphs $G = (V, E)$. We reserve n to denote the order ($|V|$) of the graph under consideration and q the size ($|E|$). A graph will be called *hamiltonian* if it contains a spanning cycle. Such a cycle will be called a *hamiltonian cycle.* If a graph G contains a spanning path it is termed a *traceable* graph and if G contains a spanning path joining any two of its vertices, then G is *hamiltonian - connected.* If G contains a cycle of each possible length l, $3 \le l \le n$, then G is said to be *pancyclic.* These are clearly closely linked ideas and by no means does this list exhaust the related concepts.

There are four fundamental results that I feel deserve special attention here; both for their contribution to the overall theory and for their affect on the development of the area. In many ways, these four results are the foundation of much of today's work.

Beginning with Dirac's Theorem [93] in 1952, the approach taken to developing sufficient conditions for a graph to be hamiltonian usually involved some sort of edge density condition; providing enough edges to overcome any obstructions to the existence of a hamiltonian cycle. Dirac saw a natural method for supplying the necessary edges, using the minimum degree $\delta(G)$.

Theorem 0.1 [93]. If G is a graph of order n such that $\delta(G) \ge \frac{n}{2}$, then G is hamiltonian.

Dirac's Theorem was followed by that of Ore [242]. Ore's Theorem relaxed Dirac's condition and extended the methods for controlling the degrees of the vertices in the graph.

Theorem 0.2 [242]. If G is a graph of order n such that $deg\ x + deg\ y \geq n$, for every pair of nonadjacent vertices $x, y \in V$, then G is hamiltonian.

This relaxation stimulated a string of subsequent refinements (see [70]or [37] for more details), culminating in the classic work of Bondy and Chvátal [51] concerning stability and closure. In [51], as in Ore's [242] motivating work, independent (mutually nonadjacent) vertices whose degree sum is at least n are fundamental. The following notation will be useful:

$$\sigma_k(G) = \min \{ \sum_{i=1}^{k} deg\ v_i \mid \{ v_1, v_2, \ldots, v_k \} \text{is independent in } G \ (k \geq 2) \}.$$

In [51], Bondy and Chvátal extended Ore's Theorem in a very useful way. Define the $k-$(degree) *closure* of G, denoted $C_k(G)$, as the graph obtained by recursively joining pairs of nonadjacent vertices whose degree sum is at least k, until no such pair remains. Their fundamental hamiltonian result is the following:

Theorem 0.3 [51]. A graph G of order n is hamiltonian if, and only if, $C_n(G)$ is hamiltonian.

Theorem 0.3 provides an interesting relaxation of Ore's condition. Now we no longer need to verify that each pair of nonadjacent vertices has degree sum at least n, but rather, only enough pairs to ensure that the closure is recognizable as being hamiltonian. Since the closure is hopefully a denser graph, your chances should improve. However, the number of edges actually added in forming the degree closure can vary widely. It is easy to construct examples for all possible values from 0 to $\binom{p}{2} - q$. Thus, we might receive no help in deciding if the original graph is hamiltonian, or the degree closure may be the complete graph.

This idea led naturally to the following definition. Let $\hat{P}$ be a property defined for all graphs of order n and let k be an integer. Then $\hat{P}$ is said to be $k-$degree stable if, for all graphs G of order n, whenever $G + uv$ has property $\hat{P}$ and $deg\ u + deg\ v \geq k$, then G has property $\hat{P}$. Among the results established in [51] were the following:

i. The property of being hamiltonian is $n-$degree stable.

ii. The property of being traceable is $n - 1-$degree stable.

iii. The property of containing a C_s $(5 \leq s \leq n)$ is $(2n - 1)-$degree stable.

The fourth fundamental result took a different approach. Let $\beta_0(G)$ denote the independence number of G, that is, the size of a maximal independent set of vertices in G.

Theorem 0.4 [78]. If G is a graph with connectivity k such that $\beta_0(G) \leq k$, then G is hamiltonian.

In the following sections, we shall see that each of these results has inspired many others.

Section 1 Generalizations of the Fundamentals

Many generalizations of Theorems 0.1 - 0.4 have been found. Häggkvist and Nicoghossian [146] sharpened Dirac's Theorem by incorporating the connectivity of the graph into the degree bound.

Theorem 1.1 [146]. If G is a $2-$connected graph of order n, connectivity k and minimum degree $\delta(G) \geq \frac{1}{3}(n + k)$, then G is hamiltonian.

This result itself was recently generalized in [25].

Theorem 1.2 [25]. If G is a $2-$connected graph of order n and connectivity k such that $\sigma_3(G) \geq n + k$, then G is hamiltonian.

A natural direction, taken by Bondy [50], was to further increase the number of vertices involved in the independent set.

Theorem 1.3 [50]. If G is a $k-$connected graph of order $n \geq 3$ such that $\sigma_{k+1}(G) > ½(k + 1)(n - 1)$, then G is hamiltonian.

Degree sum conditions like those of Theorems 0.2 and 1.3 do have a major shortcoming however; they apply to very few graphs. Thus, it is natural to consider variations on such conditions, with the hope that these variations will be more applicable.

Along these same lines, Bondy and Fan [52] provided an Ore-type result for finding a *dominating cycle*, that is, a cycle that is incident to every edge of the graph. Harary and Nash-Williams [149] showed that the existence of a dominating cycle in G is essentially equivalent to the existence of a hamiltonian cycle in the line graph of G, denoted $L(G)$.

Theorem 1.4 [52]. Let G be a $k-$connected $(k \geq 2)$ graph of order n. If any $k + 1$ independent vertices x_i $(0 \leq i \leq k)$ with $N(x_i) \cap N(x_j) = \phi$ $(0 \leq i \neq j \leq k)$ satisfy $\sigma_{k+1}(G) \geq n - 2k$, then G contains a dominating cycle.

This result has the immediate Corollary that if G is $k-$connected with $\delta(G) \geq \frac{n - 2k}{k + 1}$, then G has a dominating cycle. This proves a conjecture of Clark, Colburn and Erdös [79]. Fraisse [122] had independently proved this conjecture, however, his result is slightly weaker than that of Bondy and Fan. Bondy and Fan [52] also made the following conjecture. Let $R_m(v) = \{ u \in V(G) \mid dist(u, v) \leq m \}$.

Conjecture [52]. Let G be a $k-$connected graph $(k \geq 2)$. If any $k + 1$ vertices x_i $(0 \leq i \leq k)$ with $R_m(x_i) \cap R_m(x_j) = \phi$ $(0 \leq i \neq j \leq k)$ satisfy the inequality

$$\sum_{i=0}^{k} | R_m(x_i) | \geq n - 2k,$$

then G has an $m-$dominating cycle (that is, a cycle C such that $R_m(v) \cap C \neq \phi$ for every $v \in V(G)$).

Bondy [50] also gave a sufficient condition for G to contain a cycle C with the property that $G - V(C)$ contains no clique K_k. When $k = 1$, this result corresponds to Ore's Theorem. Veldman [314] further generalized this idea. A cycle C is said to be D_λ*−cyclic* if, and only if, every connected subgraph of order λ has at least one vertex in common with C. This idea also generalizes the idea of a dominating cycle. Veldman [314] generalized Theorem 1.1 as well as others to D_λ*−cycles*.

Another very interesting approach was introduced by Fan [105]. He showed that we need not consider "all pairs of nonadjacent vertices", but only a particular subset of these pairs.

Theorem 1.5 [105]. If G is a $2-$connected graph of order n such that

$$\min\{ \max (deg\ u, deg\ v) \mid dist (u, v) = 2 \} \geq \frac{n}{2},$$

then G is hamiltonian.

Fan's Theorem is significant for several reasons. First it is a direct generalization of Dirac's Theorem. But more importantly, Fan's Theorem opened an entirely new avenue for investigation; one that incorporates some of the local structure, along with a density condition. Now, when attempting to find new adjacency results, one must not only consider the "degree bounds", but the set of vertices for which this bound applies. A natural question will be: Can an even sparser set of vertices be used (thus expanding the number of graphs for which the result will apply)? We shall see later that this idea can be used in conjunction with other adjacency conditions and that incorporating more of the structure beyond the neighborhood of a vertex can be useful.

Theorem 1.5 was strengthened in [35], where the same conditions were shown to imply the graph is pancyclic, with a few minor exceptions.

Problem.

1. Can vertices at distance three be used to produce a Fan-type result? What about larger distances?
2. Does there exist a digraph analog to Fan's Theorem?

Recently, a new "generalized degree" approach based upon neighborhood unions has proven to be useful. This idea is based on the adjacencies of a set S of vertices. The degree of a set S is defined to be

$$deg(S) = | \bigcup_{v \in S} N(v) |,$$

where $N(v) = \{ x \in V(G) \mid xv \in E(G) \}$ is the *neighborhood of v*. Typically, S is chosen to have some property $\hat{P}$ (for example, independence). This relaxation further generalizes the approach taken in the

60's and early 70's and offers a wide variety of uses.

The first use of the generalized degree condition was to provide another generalization of Dirac's Theorem.

Theorem 1.6 [108]. If G is a $2-$connected graph of order n such that $deg\ (S) \geq \frac{2n-1}{3}$ for each $S = \{x, y\}$ where x and y are independent vertices of G, then G is hamiltonian.

Fraisse [123] extended this result to larger independent sets of vertices.

Theorem 1.7 [123] Let G be a $k-$connected graph of order n. Suppose there exists some $t \leq k$, such that for every independent set S of vertices with cardinality t we have $deg\ (S) \geq \frac{t(n-1)}{t+1}$, then G is hamiltonian.

Very recently, Lindquester [200] was able to show that a Fan-type restriction to vertices at distance two could also be used with generalized degrees, providing an improvement to Theorem 1.6.

Theorem 1.8 [200]. If G is a $2-$connected graph of order n satisfying $deg\ (S) \geq \frac{2n-1}{3}$ for every set $S = \{x, y\}$ of vertices at distance 2 in G, then G is hamiltonian.

Independent sets are not the only ones that have been useful in conjunction with generalized degrees. The collection of all pairs of vertices (or all $t-$sets of vertices) provides yet another generalization of Dirac's Theorem; one with a more combinatorial flavor.

Theorem 1.9 [107]. If G is a $2-$connected graph of sufficiently large order n such that $deg\ (S) \geq \frac{n}{2}$ for every set S of two distinct vertices of G, then G is hamiltonian.

A similar result holds for sets of more than two vertices (see [107]), however, at this time the best known lower bound is $\frac{n}{2} + c(k)$ where $c(k)$ is a constant that depends upon k, the number of vertices in the set.

We should also note here that other properties can be used to help reduce the lower bound on the generalized degree. One such result is the following.

Theorem 1.10 [135]. Let G be a graph of order n. If for every set $S = \{x, y\}$ of two independent vertices in G, $deg\ (S) \geq \frac{n}{2}$ and $|N(x) \cap N(y)| \geq 3$, then G is hamiltonian.

Many other results have been discovered in the last few years using this generalized degree (neighborhood union) condition. For a survey of such results see [197].

Problem. Find directed graph analogs to the generalized degree results.

By varying the typical degree sum approach to that of adjacent vertices rather than nonadjacent vertices, Brualdi and Shaney [61] obtained a hamiltonian result about the *line graph, L(G),* of the given graph.

Theorem 1.11 [61]. If G is a graph of order $n \geq 4$ such that for any edge uv in G, $deg\ u + deg\ v \geq n$, then G contains a dominating circuit, hence $L(G)$ is hamiltonian.

Veldman [314] further developed this idea. His work can be viewed as yet another form of generalized degree. We follow his notation here. Call two subgraphs H_1 and H_2 of G *close* in G, if they are disjoint and there is an edge of G joining a vertex in H_1 and a vertex of H_2. If H_1 and H_2 are disjoint, but not close, then they are said to be *remote.* The degree of an edge e of G is the number of vertices of G close to e when e is viewed as a subgraph of order two. We denote the edge degree as $deg(e)$. Clearly, this is nearly the generalized degree of an adjacent pair of vertices.

Theorem 1.12 [314]. Let G be a $k-$connected graph ($k \geq 2$) such that for every $k + 1$ mutually remote edges $e_0, e_1, \ldots, e_k$ of G,

$$\sum_{i=0}^{k} deg\ (e_i) > \tfrac{1}{2} k(n - k)$$

then G contains a dominating cycle.

Veldman further conjectures that this bound can be improved to $\frac{1}{3}(k + 1)(n - 2)$.

In [33], this work was extended to pancyclic line graphs. Veldman also used this approach in [313].

Ainouche and Christofides [2] combined Pósa [255] and Ore [242] type conditions on degrees to obtain interesting new results. In a graph $G = (V, E)$, with $W \subseteq V$, let

$$deg\ w_1 \leq deg\ w_2 \leq \cdots \leq deg\ w_{|W|}$$

be the degrees in G of the vertices in W. A subset W of $V(G)$ is termed "good" if $deg\ w_i > i$ for every $w_i \in W$. With this in mind, Ainouche and Christofides [2] obtained the following.

Theorem 1.13 [2]. Let G be a graph of order n and W be a good subset of $V(G)$. If $deg\ x + deg\ y \geq n$ for any two nonadjacent vertices x, y in $V - W$, then G is hamiltonian.

Ainouche and Christofides also obtained descriptions of maximal nonhamiltonian graphs failing to satisfy their condition.

Dirac's condition ($\delta(G) \geq \frac{n}{2}$) implies that any $m-$regular graph of order at most $2m$ is hamiltonian. Another way of saying this is that every path of length zero (namely a vertex) is contained in a hamiltonian cycle. Ore [243] established that every $m-$regular graph of order at most $2m - 1$ is hamiltonian - connected. Tomescu ([306] and [307]) has extended this further. In [306], he shows that any $m-$regular graph of order $2m$ has the property that ant two adjacent edges are contained in a hamiltonian cycle. This implies that such graphs contain at least $\binom{m}{2}$ different hamiltonian cycles. In [306], the following was established.

Theorem 1.14 [306]. Let G be an $m-$regular graph of order $2m - k$ ($mk = 0 \mod 2$).

a. If $k = 1$, then any path of length two is contained in a hamiltonian cycle of G, (when $m \geq 3$).

b. If $k \geq 2$ and G does not contain a spanning subgraph isomorphic to $K_{m,m-k}$, then any path of length $k + 1$ is contained in a hamiltonian cycle of G, ($m \geq 2k + 1$).

In [339], it is shown that every $2-$connected $k-$regular graph G of order n is hamiltonian if $n = 3k + 1$, unless G is the Petersen graph. This answered a conjecture of Jackson. Still unsolved is the following conjecture also due to Jackson.

Conjecture. For all $k \geq 4$, all $2-$connected $k-$regular graphs of order at most $3k + 3$ are hamiltonian.

Recently, Asratyan and Khachatryan [14] introduced yet another Ore-type adjacency condition that is reminiscent of Fan's use of vertices at distance two. Let $G_2(x)$ denote the subgraph of G induced by those vertices at distance at most 2 from x.

Theorem 1.15 [14]. Let G be a graph of order n. Suppose that whenever $deg\ x \leq \frac{n-1}{2}$ and y is a vertex at distance 2 from x,

$$deg\ x + deg_{G_2(x)} y \geq |\ V(G_2(x))\ |,$$

then G is hamiltonian.

Another Ore-type result is due to Hakimi and Schmeichel [147].

Theorem 1.16 [147]. Let G be a graph of order $n \geq 3$ with a hamiltonian cycle C: $x_1, x_2, \ldots, x_n, x_1$. Suppose that $deg\ x_1 + deg\ x_n \geq n$. Then G is either

1. pancyclic,

2. bipartite, or
3. missing only an $(n-1)-$cycle.

Moreover, if case 3 occurs. they are able to provide a great deal more information on the local structure around the vertices x_1 and x_n on C.

Denote by $\omega(G)$, the number of components of a graph G. Using this parameter, Chvátal [77] introduced the following concept: We say G is $1-$tough if $\omega(G-S) \le |S|$ for every subset S of $V(G)$ with $\omega(G-S) > 1$. In general, we say that G is *t-tough* if for every vertex cut-set S, $\omega(S) \le \frac{|S|}{t}$. Chvátal showed that if G is hamiltonian, then $t \ge 1$. He also conjectured that if G was $2-$tough, then G was hamiltonian. Thomassen and others have produced examples of nonhamiltonian graphs with $t > \frac{3}{2}$. Molluzzo [224] also studied toughness. He showed that if G is hamiltonian-connected, then $t > 1$ and that this is best possible. Further, he showed that if G is $s-$hamiltonian (that is, the removal of fewer than s vertices leaves a hamiltonian graph), then $t \ge 1 + \frac{s}{\beta_0}$ (where β_0 is the independence number of G). (Note that recognizing toughness has recently been shown to be an NP - complete problem [26]).

Toughness, when combined with other conditions, can be used to obtain both new results and improvements of existing results. (See also [43] and [177].)

Theorem 1.17 [175]. Let G be a $1-$tough graph of order $n \ge 11$ such that $\sigma_2(G) \ge n-4$. Then G is hamiltonian.

Theorem 1.18 [27]. Let G be a $2-$tough graph of order n such that $\sigma_3(G) \ge n$. Then G is hamiltonian.

Further generalizations of Theorem 1.17 can be found in [287] and generalizations of Fan's Theorem with regard to toughness can be found in [24]. For a more complete survey of results relating toughness and hamiltonian properties, see [28].

Turning to work related to Theorem 0.4, we find that in [54] it was shown that a $2-$connected graph with $\beta_0(G) \le 2$ is either pancyclic, or one of the graphs C_4 or C_5. Amar, Fournier, Germa and Häggkvist [10] showed that if G is $k-$connected with $\beta_0(G) = k+1$, then for every maximum length cycle C of G, $G - V(C)$ is complete. More recently, Benhocine and Fouquet [34] considered hamiltonian line graphs in this context.

Theorem 1.19 [34]. If G is a $2-$connected graph and $\beta_0(G) \le k(G) + 1$, then $L(G)$ is pancyclic unless G is one of C_4, C_5, C_6, C_7, the Petersen graph or the graph of Figure 1.1.

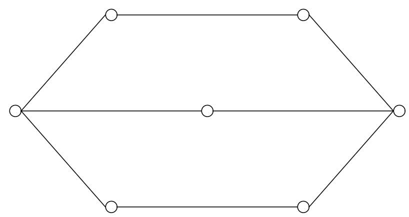


Figure 1.1 A graph whose line graph is not pancyclic.

Many results related to Theorems 0.1-0.2 have been found for digraphs. In 1981, Bermond and Thomassen [41] gave an outstanding survey of these and many other results on cycles in digraphs. I shall concentrate on subsequent work.

If D is a digraph and $S \subset V(D)$, we say that S is β_0- *independent* if the digraph induced by S, denoted $D[S]$, contains no arcs; we say that S is β_1- *independent* if $D[S]$ contains no cycles; we say that S is β_2- *independent* if $D[S]$ contains no $2-$cycles. Thus, $\beta_0 \leq \beta_1 \leq \beta_2$ and if D is the digraph obtained from a graph G by replacing each edge of G by a directed $2-$*cycle*, then $\beta_0 = \beta_1 = \beta_2$. Thus, each parameter may be considered a directed analogue of the undirected independence number β_0.

Thomassen [299] gave examples of nonhamiltonian $2-$connected digraphs with $\beta_2(D) = 2$ and non-hamiltonian $3-$connected digraphs with $\beta_1 = 3$ and $\beta_0 = 2$. Thus, the Erdös-Chvátal Theorem does not completely generalize to digraphs. The following problem was posed by Jackson [171].

Problem. Determine if for every integer m, there exists an integer (smallest) $f_i(m)$ ($i = 0, 1,$ or 2) such that every $f_i(m)-$connected digraph D with $\beta_i(D) \leq m$ is hamiltonian.

Jackson [171] and Jackson and Ordaz [172] have investigated this problem.

Theorem 1.20 [171].

1. Let D be a digraph with $\beta_2(G) \leq r$. If $k(D) \geq 2^r(r+2)!$, then D is hamiltonian.
2. Let D be a digraph such that $V(D)$ can be covered with m complete symmetric subgraphs. If $k(D) > m(m-1)$, then D is hamiltonian.

A digraph is said to be *2-cyclic* if any two of its vertices are contained in a common cycle.

Theorem 1.21 [172]. If D is a $k-$connected digraph and

1. if $k \geq 2\beta_1(D) - 1$, then D is $2-$cyclic,
2. if $k \geq 3$, and $\beta_0(D) \leq 2$, then D is $2-$cyclic,
3. if $k \geq 15$ and $\beta_0(D) \leq 3$, then D is $2-$cyclic,

4. if $k \geq 1$ and $\beta_0(D) = 1$, then D contains cycles of length l for $3 \leq l \leq n$.
5. if $k \geq 3$ and $\beta_2(D) \leq 2$, then D contains cycles of all lengths l, $2 \leq l \leq n$.

Jackson and Ordaz [172] also posed several more problems.

Problem.

1. Does there exist an integer k such that every $k-$connected digraph D with $\beta_0(D) = 2$ is hamiltonian?
2. Does every $k-$connected digraph D with $\beta_0(D) \leq k + 1$ have a hamiltonian path?

Conjecture [172]. Given any integer m, there exists a smallest integer $g(m)$ such that every $g(m)-$connected digraph D with $\beta_0(D) \leq m$ is $2-$cyclic.

Section 2 Random Graphs and the Use of Probability

A large part of the difficulty in finding an effective characterization for hamiltonian graphs certainly stems from the fact that so many graphs are hamiltonian. Yet, if so many graphs are hamiltonian, we should be able to say something more about what we mean by a property being "very common" among graphs. In order to be more precise here, probabilistic methods will be helpful. It is not my purpose to introduce the reader to random graph techniques. However, I shall try to define enough to hopefully make the results of this section understandable to those not familiar with this subject.

We shall use $Pr(X)$ to denote the probability of event X. If Ω_n is a model of random graphs of order n, we say *almost every* graph in Ω_n has property Q if $Pr(Q) \rightarrow 1$ as $n \rightarrow \infty$. Note that this is equivalent to saying that the proportion of all labeled graphs of order n that have Q tends to 1 as $n \rightarrow \infty$.

In their classic paper on the evolution of random graphs, Erdös and Rényi [102] posed the following questions.

— In what models for random graphs is it true that almost every graph is hamiltonian?

— How large does $q = q(n)$ have to be to ensure that almost every random n vertex q edge graph is hamiltonian?

There are two fundamental models for defining probability measures on the set of all 2^M subgraphs (here $M = \binom{n}{2}$) of an n vertex complete graph. Both of these models have been extensively studied.

- (The edge density model) Suppose that $0 \leq p \leq 1$. Let $G_{n,p}$ denote a graph on n vertices obtained by inserting any of the M possible edges with probability p.
- (The fixed size model) Suppose that $N = N(n)$ is a prescribed function of n which takes on values in the set of positive integers. Then there are $S = \binom{M}{N}$ different graphs with N edges possible on

the vertex set $\{ 1, 2, \ldots, n \}$. We let $G_{n,N}$ denote one of these graphs chosen uniformly at random with probability $\frac{1}{S}$.

Although some preliminary results concerning hamiltonian properties of random graphs were obtained in the early 70's, the first major advance in this area was achieved independently by Pósa [256] and Korshunov [189], when they proved the following result.

Theorem 2.1 [256],[189]. There exists a constant c such that almost every labeled graph on n vertices and at least $cn \log n$ edges is hamiltonian.

A property $\hat{P}$ is called *monotone* if whenever G has property $\hat{P}$ and $G \subseteq H$, then H has property $\hat{P}$. Clearly, the property of being hamiltonian is monotone. Erdös and Rényi noticed an important and interesting fact about most monotone properties - they appear suddenly. By this we mean that for some $M = M(n)$, almost no $G_{n,M}$ has property $\hat{P}$, while for a slightly larger M, almost every $G_{n,M}$ has property $\hat{P}$. The property of being hamiltonian behaves in this manner.

To be more specific, given a monotone increasing property, a function $M^*(n)$ is said to be a threshold function for $\hat{P}$ if

$$\frac{M(n)}{M^*(n)} \to 0 \text{ implies that almost no } G_{n,M} \text{ has } \hat{P}, \text{ and}$$

$$\frac{M(n)}{M^*(n)} \to \infty \text{ implies that almost every } G_{n,M} \text{ has } \hat{P}.$$

Hence, a threshold function describes a critical time, before which $\hat{P}$ is highly unlikely and after which it is extremely likely.

It should be clear that threshold functions are not unique, however, they are unique up to factors. That is, given two threshold functions for $\hat{P}$, say M_1^* and M_2^*, then $M_1^* = O(M_2^*)$ and $M_2^* = O(M_1^*)$. Thus, we may speak of the threshold function of $\hat{P}$.

It is also clear that if G is a hamiltonian graph, then its minimum degree $\delta(G) \geq 2$. Thus, we see that

$$Pr(\ G_{n,M} \text{ is hamiltonian }) \leq Pr(\ \delta(G_{n,M}) \geq 2\ ).$$

Komlos and Szemeredi [188] and Korshunov [190] were the first to link the threshold for $\delta(G) \geq 2$ with the threshold for G being hamiltonian. It was known that

$$Pr(\ \delta(G_{n,M}) \geq 2\ ) \to 1 \text{ if, and only if, } \omega(n) = \frac{2M}{n} - \log n - \log\log n \to \infty.$$

They showed that this necessary condition was also sufficient to ensure that almost every $G_{n,\,M}$ and $G_{n,\,p}$ is hamiltonian.

Theorem 2.3 [188],[190]. Suppose $\omega(n) \to \infty$ as $n \to \infty$, and let $p = \frac{1}{n}\{ \log n + \log\log n + \omega(n) \}$ and $M(n) = \left\lfloor \frac{n}{2}\{ \log n + \log\log n + \omega(n) \} \right\rfloor$.

Then almost every $G_{n,p}$ is hamiltonian and almost every $G_{n,M}$ is hamiltonian.

In fact, they showed an even more direct relationship.

Theorem 2.4 [188],[190]. Assume that a random labeled graph is constructed as follows: the first edge is chosen at random, the second edge is chosen at random from the remaining $\binom{n}{2} - 1$ possibilities, etc., until a graph with minimum degree 2 is formed. Then the probability that the resulting graph is hamiltonian approaches 1 as $n \rightarrow \infty$.

Theorem 2.4 provides us with an "almost sure decision rule" to decide if a graph is hamiltonian: Simply check whether it contains vertices of degree 0 or 1. The number of times we will be wrong is negligible for large n.

Further improvements were made by Shamir [273], Bollobás, Fenner and Freize [48] and Freize [124]. The algorithmic aspects of these improvements will be discussed in Section 4.

Theorem 2.5 [273].

i. Let $p = \frac{1}{n}(\log n + c \log \log n)$, $c > 3$. Then almost every graph in $G_{n,p}$ contains a hamiltonian path.

ii. If $M(n) = \frac{n}{2}(\log n + (4 + \varepsilon) \log \log n)$, $\varepsilon > 0$, then almost every $G_{n,M}$ is hamiltonian.

Bollobás, Fenner and Frieze [48] used the following strengthening of Theorem 2.3 due to Komlos and Szemeredi [188] to produce their algorithmic work.

Theorem 2.6 [188]. For $M(n) = \frac{n}{2}(\log n + \log \log n + c_n)$

$$\lim_{n \rightarrow \infty} Pr(G_{n,M} \text{ is hamiltonian}) = \begin{cases} 0 & \text{if } c_n \rightarrow -\infty \\ e^{-e^{-c}} & \text{if } c_n \rightarrow c \\ 1 & \text{if } c_n \rightarrow \infty. \end{cases}$$

For $V_n = \{1, 2, \ldots, n\}$, let $v \in V_n$ independently make m random (but not necessarily distinct) choices $c(v, i) \in V_n$, $i = 1, 2, \ldots, n$. This is done independently for each $v \in V_n$. Then consider the multigraph

$$D(n, m) = (V_n, E(n, m)), \text{ where}$$

$$E(n, m) = \{ (v, c(v, i)) \mid v \in V_n, 1 \le i \le m, \text{ and } v \ne c(v, i) \}.$$

(That is, we ignore the orientation on the edges $(v, c(v, i))$, but we do not coalesce multiple edges or remove loops. Then with this in mind, the following results were obtained:

Theorem 2.7 [113]. For $m \geq 23$, $\lim_{n \to \infty} Pr(\ D(n, m) \text{ is hamiltonian }) = 1$.

They further conjecture the naturally anticipated fact that this can be improved to $m \geq 3$. Freize [124] was able to improve this to $m \geq 10$ as well as improve the time of the algorithm used to produce the cycle (see Section 4 for more details).

Let $R(n, r)$ denote the random regular graph chosen uniformly from the set of $r-$regular graphs on V_n. Bollobás [46] and Fenner and Freize [114] independently proved that there is a constant r_0 such that for any $r \geq r_0$,

$$\lim_{n \to \infty} Pr(\ R(n, r) \text{ is hamiltonian }) = 1.$$

In [114], it was shown that $r_0 = 796$, while in [124], this was improved to $r_0 = 85$. Again, Freize conjectures that the best value actually is $r_0 = 3$.

One might hope that the problem of finding hamiltonian cycles in random bipartite graphs is easier then in $G_{n,p}$. However, this is not the case. Progress was made by Freize [125]. Here we let $G_{n,n;p}$ denote a random bipartite graph with n vertices in each partite set and probability p that any edge is in $G_{n,n;p}$.

Theorem 2.8 [125]. Let $p = \left(\dfrac{\log n + \log\log n + c_n}{n} \right)$. Then the probability that $G_{n,n;p}$ is hamiltonian tends to $e^{-2c^{-c}}$ as $c_n \to c$.

As with random graphs, the obstacle to be overcome in random bipartite graphs turns out to be the existence of vertices of degree at most 1.

Turning to digraphs, we note that the analogous problem seems harder, especially in view of the fact that the useful work of Pósa [256] (see Section 4 for more details) does not have directed analogues. But despite this problem, McDiarmid [218,219] was able to show that the probability that a random digraph $D_{n,p}$ is hamiltonian is not smaller than the probability that $G_{n,p}$ is hamiltonian. Using this fact he deduced the following result.

Theorem 2.9 [218,219]. If $p = \frac{1}{n}(1 + \varepsilon)(\log n)$ then

$$Pr(\ D_{n,p} \text{ is hamiltonian }) \to \begin{cases} 1, & \text{if } \varepsilon > 0 \\ 0, & \text{if } \varepsilon < 0. \end{cases}$$

Next we turn our attention to regular graphs. Since vertices of degree at most 1 have been the fundamental obstruction to hamiltonian cycles in general random graphs, we have been forced to produce enough edges to ensure that we overcome this difficulty. It seems reasonable to hope that the edge density can be lessened by overcoming this difficulty in other ways, namely, by requiring that G be $r-$regular for some $r \geq 2$.

Following Bollobás [47], we consider the class of graphs $\mathcal{G}(n, k-out)$; formed with vertex set $V = \{1, 2, \ldots, n\}$ and for each vertex $x \in V$, select k other vertices (with all $\binom{n-1}{k}$ choices equally likely), and for each selected y, direct an edge from x to y. Let $\vec{D}$ be a random directed graph formed in this way. Let G_{k-out} be the random graph with vertex set V and edge set

$$\{ xy \mid \text{at least one of } \vec{xy} \text{ and } \vec{yx} \text{ is an edge of } \vec{D} \}.$$

We denote by $\mathcal{G}_{k-out}$ the collection of all graphs G_{k-out}. Since for a fixed k, the graphs in $\mathcal{G}_{k-out}$ have only $O(n)$ edges, we are now looking for hamiltonian cycles in truly sparse graphs. Fenner and Freize [113] accomplished a major step when they verified these graphs are almost always hamiltonian. Their proof was the first example of the "coloring technique" that has proved most useful in this area.

Theorem 2.10 [113]. There is a natural number k_0 such that if $k \geq k_0$, then almost every G_{k-out} is hamiltonian.

In view of Theorem 2.10, it is not surprising that if r is sufficiently large, almost every random $r-$regular graph is hamiltonian. This was shown independently by Bollobás [44] and Fenner and Freize [114].

Another development that allows us to sometimes be more precise in determining thresholds is the following: A random graph process on $V = \{1, 2, \ldots, n\}$ is a Markov chain $\tilde{G} = (G_t)_o^\infty$, whose states are graphs on V. The process starts with an empty graph and for $1 \leq t \leq \binom{n}{2}$, the graph G_t is obtained from G_{t-1} by the addition of a single edge, with all new edges being equiprobable. Thus, G_t has exactly t edges, unless $t > \binom{n}{2}$, in which case we define $G_t \simeq K_n$.

If G is the set of all $N!$ graph processes, then G can be made into a probability space by assuming all processes are equally likely. Then almost every graph process $\tilde{G}$ is said to have property $\hat{P}$ if the probability that $\tilde{G}$ has property $\hat{P}$ tends to 1 as $N \to \infty$. The hitting time, τ, of a monotone property $\hat{P}$ is defined to be

$$\tau(\tilde{G};\ \hat{P}) = \min\{ t \geq 0 \mid G_t \text{ has } \hat{P} \}$$

Bollobás [45] established the tie between minimum degree and hamiltonian cycles for graph processes. Naturally, it involves the hitting time of $\delta \geq 2$.

Theorem 2.11 [45]. Almost every graph process $\tilde{G}$ is such that

$$\tau(\tilde{G};\ \text{hamiltonian}) = \tau(\tilde{G};\ \delta \geq 2).$$

Other interesting results along these lines are due to Robinson and Wormald [261] who proved that the probability that a cubic graph is hamiltonian is at least 0.974. They also showed that almost every cubic bipartite graph is hamiltonian. However, Richmond, Robinson and Wormald [259] showed that at times hamiltonian cycles are rare.

Theorem 2.12 [259]. Almost every cubic planar graph is nonhamiltonian.

Section 3 Forbidden Subgraphs

A new approach to the hamiltonian problem, although not new to Graph Theory in general, began with a rather innocent observation due to Goodman and Hedetniemi [131]. Before exploring this approach, some terms will be helpful. Given graphs $F_1, F_2, \ldots, F_k$, we say that G is $\{ F_1, F_2, \ldots, F_k \}$ – free if G contains no induced subgraph isomorphic to any F_i, $(1 \leq i \leq k)$.

In considering graphs that are free of some set of graphs, we are restricting our attention to a class of graphs defined with specific structural limitations. Thus, we may be able to avoid the pure density type arguments seen earlier. Our hope of course, is to find conditions that will work on graphs not previously covered by density results. In fact, what we tend to obtain are results that apply when the graphs are either dense or very sparse.

Central to most forbidden subgraph results to date is the complete bipartite graph $K_{1,3}$ (sometimes called a *claw)* or graphs very closely related to $K_{1,3}$ (see Figure 3.1). Some other graphs that have proven to be useful are shown in Figure 3.2.

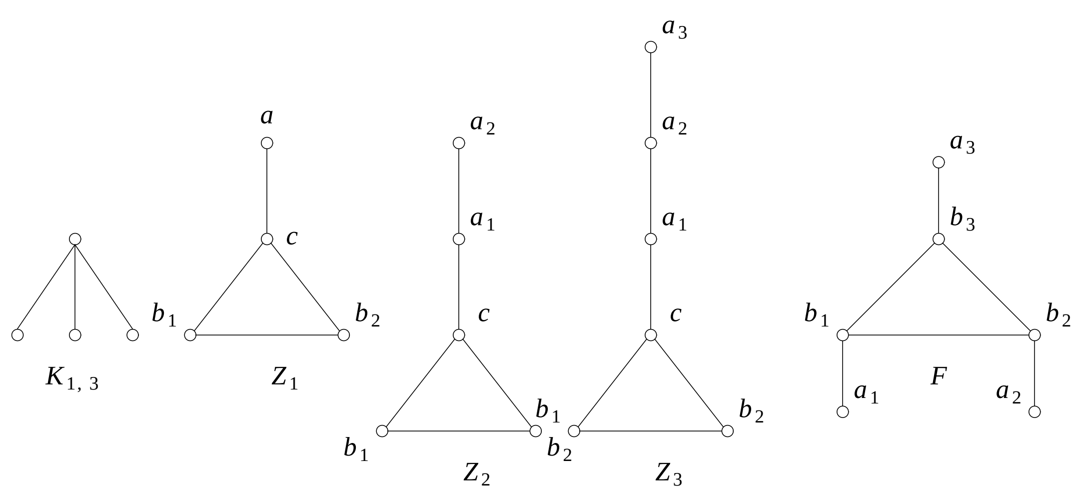


Figure 3.1 Graphs related to $K_{1,3}$.

We are now ready ready to state Goodman and Hedetniemi’s result.

Theorem 3.1 [131]. If G is a 2 – connected $\{ K_{1,3}, Z_1 \}$ – free graph, then G is hamiltonian.

The proof of Theorem 3.1 is very simple and in fact, it is easy to show that the only graphs satisfying its hypothesis are complete graphs, complete graphs with a matching removed or a cycle. Goodman and Hedetniemi pointed out that this seemed to be the first result that actually applied to a cycle.

In 1979, Oberly and Sumner [238] really opened the door to this approach, by relating forbidden subgraphs with another property, local connectivity. We say a graph G is locally connected, if for each vertex x, the subgraph of G induced by x is a connected graph.

Theorem 3.2 [238]. A connected, locally connected, $K_{1,3}-$free graph of order $n \geq 3$ is hamiltonian.

Further, Oberly and Sumner made several interesting conjectures.

Conjecture: If G is a connected, locally $k-$connected, $K_{1,k+2}-$free graph of order $n \geq 3$, then G is hamiltonian.

They further conjectured an even more optimistic result.

Conjecture: If G is a connected, locally $k-$connected, $K_{1,k+1}-$free graph of order $n \geq 3$, then G is hamiltonian.

They also posed the problem: Is every connected, locally hamiltonian graph hamiltonian? An affirmative answer to this problem would have produced an easy proof of the Conjectures. However, this problem was answered negatively in [248].

The work of Oberly and Sumner spurred further investigations of the same type. Attempts were made to broaden the sets of graphs that were forbidden. See Figures 3.1 and 3.2 for some of the graphs that have been used.

Theorem 3.3 [94]. Let G be a graph of order $n \geq 3$ that is $\{ K_{1,3}, F \} -$ free. Then,

i. if G is connected, then G is traceable,

ii. if G is $2-$connected, then G is hamiltonian.

This result was followed by other extensions of Theorem 3.1.

Theorem 3.4 [133]. If G is a $2-$connected $\{ K_{1,3}, Z_2 \} -$free graph, then either G is pancyclic or G is a cycle.

Since I and A are induced subgraphs of F, every $I-$free or $A-$free graph is also $F-$free. Thus, the following Corollary of Theorem 3.3 is obtained.

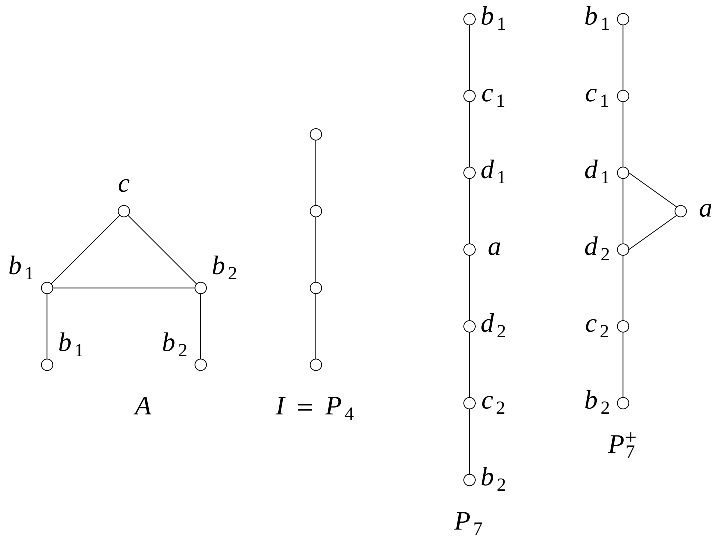


Figure 3.2   Important Forbidden Subgraphs.

Corollary 3.5 [133]. Let G be a $2-$connected $K_{1,3}-$free graph.

i. If G is $I-$free, then G is hamiltonian.

ii. If G is $A-$free, then G is hamiltonian.

Zhang [337] considered degree sums in claw free graphs. In particular, he showed that if G is a $k-$connected, $K_{1,3}-$free graph of order n such that $\sigma_{k+1}(G) \geq n - k$, then G is hamiltonian.

Broersma and Veldman [59] introduced a relaxation of the forbidden subgraph condition by allowing certain of the forbidden graphs to exist, provided their adjacencies outside their own vertex set are of the "proper type". We say a subgraph H of G satisfies property $\phi(u, v)$ if

$$(N(u) \cap N(v)) - V(H) \neq \emptyset.$$

That is, $u, v \in V(H)$ and u and v have a common neighbor in G outside of H. Using this idea, they obtained generalizations to several results, including Theorem 3.1. The vertices a, b_1 and b_2 are as in Figure 3.1.

Theorem 3.6 [59]. Let G be a $2-$connected $K_{1,3}-$free graph.

i. If every induced Z_1 of G satisfies $\phi(a, b_1)$ or $\phi(a, b_2)$, then either G is pancyclic or G is a cycle.

ii. If every induced Z_2 of G satisfies $\phi(a_1, b_1)$ or $\phi(a_1, b_2)$, then either G is pancyclic or G is a cycle.

The nonhamiltonian $K_{1,3}-$free graph of Figure 3.3 has the property that every induced Z_2 satisfies $\phi(a_1, b_1)$ or $\phi(a_1, b_2)$; hence, in Theorem 3.6, "and" cannot be replaced by "or". Broersma and Veldman also obtained a generalization of Corollary 3.5i using these ideas. They also used some other related graphs (see Figure 3.2) to obtain the following result.

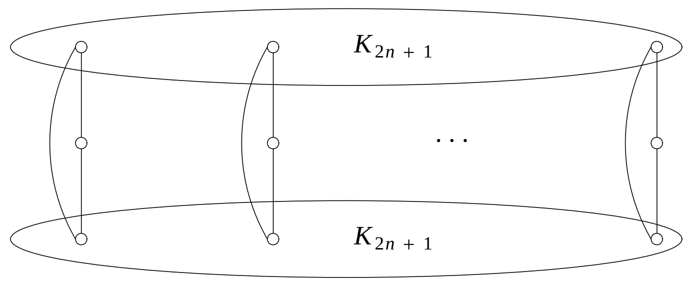


Figure 3.3 A nonhamiltonian $K_{1,3}$ − free graph.

Theorem 3.7 [59]. Let G be a 2 − connected $K_{1,3}$ − free graph. If every induced subgraph of G isomorphic to P_7 or P_7^+ satisfies $\phi(a, b_1)$ or $\phi(a, b_2)$ or ($\phi(a, c_1)$ and $\phi(a, c_2)$), then G is hamiltonian.

An immediate Corollary of Theorem 3.7 was originally obtained in [132].

Corollary 3.8 [132]. If G is a 2 − connected $K_{1,3}$ − free graph of diameter at most 2, then G is hamiltonian.

Broersma and Veldman [59] conjecture the following generalization of Corollary 3.5ii and Theorem 3.3.

Conjecture.

1. Let G be a 2 − connected $K_{1,3}$ − free graph. If every induced A of G satisfies $\phi(a_1, a_2)$, then G is hamiltonian.

2. Let G be a 2 − connected $K_{1,3}$ − free graph. If every induced F of G satisfies ($\phi(a_1, a_2)$ and $\phi(a_1, a_3)$) or ($\phi(a_1, a_2)$ and $\phi(a_2, a_3)$) or ($\phi(a_1, a_3)$ and $\phi(a_2, a_3)$), then G is hamiltonian.

Recently, a different relaxation has been explored by Flandrin and Li [117] in which they showed that if a graph does not contain "too many" claws, then it is hamiltonian.

Theorem 3.9 [117]. Let G be a 2 − connected graph of order $n \geq 16$ and minimum degree δ. If $\delta \geq \frac{n}{3}$ and if for any two nonadjacent vertices u and v, the number of induced subgraphs isomorphic to $K_{1,3}$ containing u and v is less than $\delta - 1$, then G is hamiltonian.

In [118], Flandrin and Li showed that if G is 2 − connected and $\sigma_3(G) \geq \frac{4n}{3} + \lvert N(u) \cap N(v) \cap N(w) \rvert$ then G is hamiltonian. This bound was reduced to

$n + | N(u) \cap N(v) \cap N(w) |$ in [116].

Matthews and Sumner [215,216] studied hamiltonian properties of graphs obtained from $K_{1,3}-$free graphs.

Theorem 3.10 [215,216]. Let G be a connected, $K_{1,3}-$*free* graph, then

i. G^2 is vertex pancyclic,

ii. the total graph of G is hamiltonian,

iii. if G is noncomplete, then $k(G) = 2t(G)$,

iv. if G is $3-$connected of order ≤ 20, then G is hamiltonian.

v. [216] if G is $2-$connected and $\delta(G) \geq \frac{(n-2)}{3}$, then G is hamiltonian.

Part (4) of the above Theorem, when viewed in conjunction with Chvátal's original toughness result, inspired Matthews and Sumner to make the following conjecture.

Conjecture. [215] If G is a $4-$connected $K_{1,3}-$free graph, then G is hamiltonian.

It is interesting to note that we can reduce the connectivity from 4 to 2, when we have a reasonable neighborhood union condition present.

Theorem 3.11 [109]. If G is a $2-$connected $K_{1,3}$-free graph of order $p \geq 14$ and $S = \{ x, y \}$, where x and y are nonadjacent vertices of G, and for each such S:

i. $deg\ S > \frac{(2n-2)}{3}$, then G is pancyclic,

ii. $deg\ S > \frac{(2n-3)}{3}$, then G is hamiltonian,

iii. $deg\ S > \frac{(2n-4)}{3}$, and G is connected, then G is traceable,

iv. $deg\ S > \frac{(2n-5)}{3}$ and G is $3-$connected, then G is homogeneously traceable.

Conjecture [109] : If G is a $3-$connected $K_{1,3}$-free graph of order n such that $deg\ S > \frac{(2n-5)}{3}$, where S is any set of two nonadjacent vertices, then G is hamiltonian.

Another problem in this area arose from consideration of the famous result of Fleischner [119], that the square of any two connected graph is hamiltonian. The typical example that shows that the connectivity cannot be lowered in Fleischner's Theorem is provided by $S(K_{1,3})$, the subdivision graph of the claw (see Figure 3.4), whose square is not hamiltonian.

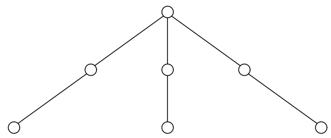


Figure 3.4 $S(K_{1,3})$, whose square is not hamiltonian.

In [134], it was conjectured that the square of any connected, $S(K_{1,3})-$free graph must be hamiltonian. This conjecture was verified by Hendry and Vogler [157]. In fact, they were able to show more.

Theorem 3.12 [157]. If G is a connected, $S(K_{1,3})-$free graph, then G is vertex pancyclic (i.e., every vertex lies on a cycle of each length l, $3 \leq l \leq n$).

Section 4 Algorithms

Despite the fact that the hamiltonian problem is NP - complete, algorithms of a probabilistic nature and algorithms for special classes of graphs have been developed.

As was mentioned in Section 2, Pósa [256] was the first to suggest an algorithm that converges almost surely for a graph of order n and size $cn \log n$, $c \geq 3$. The ideas behind his theoretic work suggested a probabilistic algorithm for determining the existence of a hamiltonian cycle. Tests of this algorithm were first performed by McGregor [see 182] on graphs of order up to 500 and by Thompson and Singhal [304] on graphs of order up to 1000. The ideas behind Pósa's work have been refined in [48] and [124] to obtain improvements in time complexity. Here we naturally only consider graphs with minimum degree at least 2.

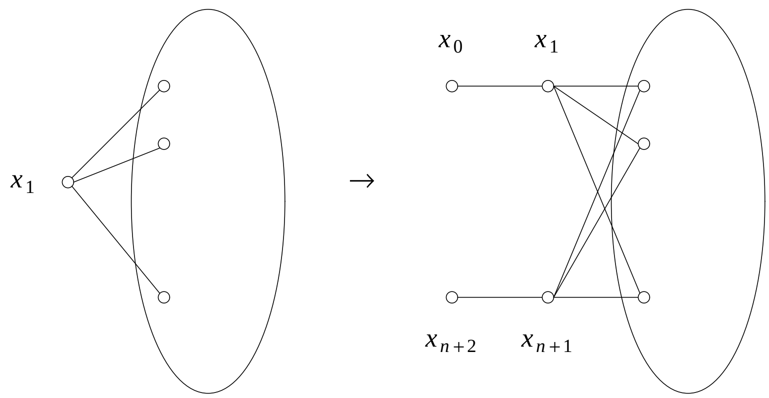
Before continuing, we wish to note that the problem of finding a hamiltonian cycle in a graph G of order n can be transformed into one of finding a hamiltonian path in a graph of order $n + 3$. This can be seen as follows:

1. Select any vertex x_1 in G.
2. Create a new vertex x_{n+1} and symmetrize x_{n+1} to x_1, that is, make x_{n+1} adjacent to exactly the same vertices as x_1.
3. Create a new vertex x_0 and make it adjacent only to x_1.
4. Create a new vertex x_{n+2} and make it adjacent only to x_{n+1}.
5. Call this new graph G^*.

Figure 4.1 The transformation to G^*.

Now it is easy to see that G has a hamiltonian cycle if, and only if, G^* has a hamiltonian path from x_0 to x_{n+2}. Thus, we shall limit our discussions to finding hamiltonian paths in graphs.

The fundamental idea behind Pósa's algorithm is a path transformation operation often called a *rotation.* It works as follows: Given a path $P = v_1, v_2, \ldots, v_k$ and an additional edge $e = v_k v_i$ ($1 \leq i \leq k-2$), we can create a new path, also of length $k-1$, by deleting the edge $v_i v_{i+1}$ and



inserting the edge *e*. Thus, define the path operation $ROTATE(\ P,\ e\ )$ as,

$$ROTATE(\ P,\ e\ ) \ = \ v_1, v_2, \ldots, v_i, v_k, v_{k-1}, \ldots, v_{i+1}.$$

The operation, *ROTATE* produces a new path with v_1 as its initial vertex and v_{i+1} as its end vertex.

Pó sa's Algorithm begins by selecting x_0 and trying to extend this trivial path, call it *P*, by including any unused neighbor of the end vertex (namely, x_0) of this path. At first this extension adds x_1 to *P*. We now repeat this step from x_1 and continue extending *P* from the non-fixed end vertex until we can no longer extend the path. At this point, either we have a hamiltonian path in G^* and we stop, or we *ROTATE* from the non-fixed end vertex of the path. Since $\delta(G) \geq 2$, we see that there must exist an edge $e = v_k v_i$ $(1 \leq i \leq k)$ and hence we can perform $ROTATE(\ P,\ e\ )$ to obtain a new path, say P'. We now try to extend this new path, rotating when we are unable to extend the (nonhamiltonian) path. We continue this process until a hamiltonian path is found or until the number of rotations exceeds some specified limit. This technique has come to be called the *extension-rotation* approach.

Pó sa's Extension-Rotation Algorithm.

1. Choose the start vertex v_0 and set $i = 0$. Set the rotation limit (*RLIMIT*) to the desired value and the rotation counter (*RCT*) to 0. Set the path length (l) to 0.

2. Choose at random any *unmarked* (that is, not previously used) neighbor *j* of the end vertex *i* ($\neq n$ unless $l = n - 2$)

   If none is found

   Then Choose at random any *marked* neighbor of *i*. Then *ROTATE* the path *P* and set $RCT \leftarrow RCT + 1$.

   If $RCT \geq RLIMIT$

   Then HALT - The algorithm has failed to find a path.

   Else mark *j* as used and set $l \leftarrow l + 1$

If $l = n$

Then HALT - A path has been found

Else Go To 1

Other early algorithms were due to Angluin and Valiant [11] and Shamir [273]. Then in 1984, Bollobás, Fenner and Freize [48] developed then first "good" algorithm for finding hamiltonian paths. Their algorithm almost always succeeded and had time complexity $O(n^{4 + \varepsilon})$. It was still based on the extension-rotation technique. Recently, Freize [124] has shown that a careful modification of Pósa's techniques can be used to produce a $O(n^3 \log n)$ time algorithm $HAM1$ which satisfies

$$\lim_{n \to \infty} Pr\ (\ HAM1 \text{ finds a hamiltonian cycle in } D(n,\ 10)\ ) = 1.$$

Further, Luczak and Freize (see [124]) have reduced the 10 above to 5.

Freize [124] also shows that there is an $O(n^3 \log n)$ time algorithm $HAM2$ which satisfies

$$\lim_{n \to \infty} Pr\ (\ HAM2 \text{ finds a hamiltonian cycle in } R(n,\ r)\ ) = 1.$$

for any constant $r \geq 85$.

Freize's improvement centers on two points. In trying to extend the path P_k, if we fail to extend, but the edge $v_0 v_k$ exists, then we know by the connectivity of the graph, that a longer path exists. Failing this, a sequence of rotations is performed in a "depth-first" manner. That is, suppose that $P_k\colon v_0, v_1, \ldots, v_k$ is the current path and that v_k has neighbors $v_{i_1}, v_{i_2}, \ldots, v_{i_j}$ on P_k. Then we replace P_k with $ROTATE(P_k, v_k v_{i_1})$ and continue our efforts with this new path before we consider $ROTATE(P_k,\ v_k v_{i_2})$, which will be done after failing to be able to extend $ROTATE(P_k,\ v_k v_{i_1})$ and backtracking. All of this is perfectly natural in the context of this problem. But Freize adds an unusual twist. He partitions $E(G)$ into two sets, E^+ and E^-. The edges of E^- are only used to close P_k to a cycle. This added condition gives him the strength to produce a $O(n^3 \log n)$ time algorithm that almost surely produces a hamiltonian path.

Another completely different recent development is due to Guravitch and Shelah [141]. They use an edge coloring based algorithm to almost always construct a hamiltonian path from a fixed initial vertex to a fixed final vertex in $\frac{cn}{p} + o(n)$ time, where c is an absolute constant and $p > \frac{3 log\ n}{n}$ is the probability that an edge exists in $G \in G_{n,\,p}$.

Their complete algorithm actually consists of three separate algorithms. The first (HPA1) almost always succeeds in $\frac{cn}{p} + o(n)$ time. When this fails, the second (HPA2) is tried and finally, if necessary, the third (HPA3) is tried. We shall look closely only at their first algorithm. We assume that the edges of G are assigned a subset of up to four colors (say red, yellow, blue and green) with certain probabilities.

The Guravitch and Shelah [141] algorithm HPA1 proceeds in stages.

Stage 0: Create four lists of vertices.

1. P_E, a path, initially consisting of only the start vertex.
2. P_O, a path, initially consisting of only the finish vertex.
3. E_O, consisting of the remaining even vertices.
4. O_O, consisting of the remaining odd vertices.

Stage 1: We extend P_E by means of successive sweeps through E_O. During one sweep, we sequentially examine the vertices in E_O. If the last vertex on P_E is adjacent to the present vertex x of E_O via a red edge, then x becomes the last vertex of the path P_E and x is removed from E_O. Halt when a sweep through E_O produces no additions to the path P_E. If E_O contains at least $\sqrt{n}$ vertices, then we go to algorithm HPA2.

This process is now repeated for P_O and O_O except that the additions are made in front of P_O, keeping the finish vertex at the end.

Stage 2: We concatenate an initial segment of P_E with a final segment of P_O. This is done so as to maximize the total number of vertices on the final path. If this cannot be done "effectively", then again we go to HPA2.

Stage 3: We now attempt to insert E_O vertices into the path P formed in Stage 2. We will make one sweep through P. We use the notation $pred(x)$ to denote the predecessor of x along P, $last(P)$ to denote the final vertex on the path P and $first(P)$ to denote the initial vertex on the list (or path) P.

1. Set $x = last(P)$.
2. If E_O is empty then HALT

   Else set $v = first(E_O)$
3. If x is one of the first 4 vertices in P then Go To HPA2.
4. If both of the edges v to $PRED(x)$ and v to x are red

   then insert v between x and $PRED(x)$ on P and set $x = PRED(x)$. Now remove v from E_O and Go To 2.

   Else If the edge from v to $PRED(x)$ is red, then set $x = PRED(x)$ and Go to 3.

   Else Set $x = PRED(x)$ and Go To 3.

Now repeat this process for the vertices in O_O.

The interesting fact about this process is that it almost always succeeds in creating a hamiltonian path, and the extra speed is gained from the fact that only the red edges are ever used in creating this path.

In the rare event that this process fails, algorithm HPA2 then tries to construct the path, using edges colored red, yellow, blue and green. This algorithm requires $O(n^2)$ time and we shall not discuss it in detail here. If further failure is encountered, HPA3 is attempted. Luckily, it is very rarely needed.

Other special case algorithms can be found in [1], [12], [267] and [272].

Section 5 Multiple Hamiltonian Cycles

In trying to construct hamiltonian graphs, it is common to notice that in the transformation from a nonhamiltonian graph to a hamiltonian graph, often many different spanning cycles are created. Thus, at times we wish to count the number of distinct cycles that are present and at other times we wish to show the existence of several edge - disjoint cycles. We shall now consider both of these questions.

We begin with results on edge disjoint hamiltonian cycles. One of the first such results is due to Nebesky and Wisztova [233] and concerns powers of graphs.

Theorem 5.1 [233]. If G is a graph of order at least $n \geq 6$ then there exists a hamiltonian cycle C of G^3 and a hamiltonian cycle C_1 of G^5 such that C and C_1 are edge - disjoint.

This result strengthens the well - known results that G^3 is hamiltonian and if $n \geq 5$, that G^5 has a $4-$factor.

Other density conditions have been developed along the lines we investigated in Section 1. Nash - Williams [232] generalized Dirac's Theorem to obtain a result on multiple edge - disjoint hamiltonian cycles.

Theorem 5.2 [232]. If G is a graph of order n such that $\delta(G) \geq \frac{n}{2}$, then G contains $\left\lfloor \frac{5(n + a_n + 10)}{224} \right\rfloor$ edge - disjoint hamiltonian cycles, where

$$a_n = \begin{cases} 0 & \text{if } n \text{ is even,} \\ 1 & \text{otherwise.} \end{cases}$$

Jackson [166] investigated multiple hamiltonian cycles in regular graphs.

Theorem 5.3 [166]. If G is a $k-$regular graph of order n $(n \geq 14)$ and $k \geq \frac{n-1}{2}$, then G contains $\frac{3k - n + 1}{6}$ edge - disjoint hamiltonian cycles.

Note that Jackson's Theorem provides a strengthening of Theorem 5.2 in the case of regular graphs. Jackson also conjectured that if G is a $k-$regular graph on n vertices, where $k \geq \frac{n-1}{2}$, then G contains $\frac{k}{2}$ edge - disjoint hamiltonian cycles. That this conjecture cannot be extended to small k ($k = 4, 5$) has been shown by Zaks [326]. He demonstrated an infinite family of 4-regular, 4-connected graphs in which any two hamiltonian cycles shared at least $\frac{1}{16}$ of their edges and he demonstrated a family of 5-regular, 5-connected planar graphs without two edge - disjoint hamiltonian cycles. Such a family of 5-regular graphs was also found by Owens [244]. Owens [244] also showed the existence for every $r \geq 3$, and every $k, \ 0 \leq k \leq \frac{n}{2}$, of an $r-$regular, $r-$connected graph that contains k edge-disjoint hamiltonian cycles, but not $k + 1$ edge disjoint hamiltonian cycles.

Faudree, Rousseau and Schelp [111] developed a degree sum condition implying the existence of multiple hamiltonian cycles and in so doing produced another generalization of Ore's Theorem.

Theorem 5.4 [111]. Let G be a graph of order $n \geq 3$ and k be a positive integer. If the sum of the degrees of any pair of nonadjacent vertices is at least $n + 2k - 2$, then for n sufficiently large ($n \geq 60k^2$ will suffice), G has k edge - disjoint hamiltonian cycles.

They further conjectured that the degree sum condition could be decreased to " $\geq n$ ", if an additional minimum degree condition was imposed. It should be noted that at the same time, Li and Zhu [199] independently proved the following:

Theorem 5.5 [199]. Let G be a graph of order $n \geq 20$ and let $\delta(G) \geq 5$. If $deg\ x + deg\ y \geq n$ for any pair of nonadjacent vertices x and y, then G contains at least two edge - disjoint hamiltonian cycles.

Faudree, Rousseau and Schelp [111] were also able to generalize another of Ore's results (the $k = 1$ case below) based on the size of the graph.

Theorem 5.6 [111]. Let k be a positive integer and G a graph of order n and size $\binom{n-1}{2} + 2k$.

1. If $n \geq 6k$, then G has k edge - disjoint hamiltonian cycles.
2. If $n \geq 6k^2$, then G has k edge - disjoint cycles of length l, for any integer l in the range 3 to n.

The generalized degree condition discussed earlier has also been used to obtain a result on multiple edge - disjoint cycles. In order to do this, several additional conditions were necessary. The edge - connectivity, $k_1(G)$, of a nontrivial graph is the minimum number of edges whose removal from G results in a disconnected graph.

Theorem 5.7 [110]. Let k be a fixed positive integer. Then there is a constant $c = c(k)$ such that if G is a graph of sufficiently large order n satisfying

1. $| N(u) \cup N(v) | \geq (\frac{2n + c}{3})$ for each pair u, v of nonadjacent vertices,
2. $\delta(G) \geq 4k + 1$,
3. $k_1(G) \geq 2k$, and
4. $k_1(G - v) \geq k$ for every vertex v,

then G contains k edge - disjoint hamiltonian cycles.

Any result that supplies sufficient conditions for a graph G to contain k edge - disjoint hamiltonian cycles and is based on a generalized degree condition like (1) must have these types of added restrictions. Examples to show this are provided in [110]. However, at this time, only conditions (3) and (4) are known to be sharp.

A corresponding result using all pairs of vertices rather than nonadjacent pairs of vertices would be interesting, but at this time remains unknown. Also, extensions of Theorem 5.7 to the case of more than two vertices would be desirable.

Bondy and Häggkvist [53] developed a generalization of the well-known result of Grinberg [139].

Theorem 5.8 [53]. Let G be a 4-regular plane graph which is decomposable into edge-disjoint hamiltonian cycles C and D. Denote by $F_{11}, F_{12}, F_{21},$ and F_{22} the sets of faces of G interior to both C and D, interior to C but not D, interior to D but exterior to C and exterior to both C and D respectively. Then

$$g(F_{11}) = g(F_{22}) \quad \text{and} \quad g(F_{12}) = g(F_{21})$$

where $g: 2^F \rightarrow N$ defined by $g(X) = \sum_{f \in X} (d(f) - 2)$ where $d(f)$ is the number of edges in the boundary of f.

Note that Zaks [327] has another generalization of Grinberg's Theorem.

The question of counting the number of hamiltonian cycles has been consider in several papers. Sheehan and Wright [275] counted hamiltonian cycles in dense graphs.

Theorem 5.9 [275]. Let G be an $(n, q)-$ graph with $\Delta(G) = \beta$ and let $H(G) =$ the number of hamiltonian cycles in $\overline{G}$ and let $M = \frac{(n-1)!}{2} =$ the number of hamiltonian cycles in K_n. Then, if

$$\frac{q}{n} \rightarrow a < \infty \text{ as } n \rightarrow \infty \text{ and } \beta = o(n) \text{ then}$$

$$\frac{H(G)}{M} \rightarrow e^{-2a} \text{ as } n \rightarrow \infty.$$

Sheehan [274] also studied graphs with exactly one hamiltonian cycle.

Theorem 5.10 [274]. Let G be a graph of order n containing exactly one hamiltonian cycle. Then the maximum number of edges in G is

$$\frac{n^2}{4} + 1.$$

As usual, special classes of graphs also provide us with a chance to say more.

Theorem 5.11 [148].

1. For all $n \geq 12$, there exists a maximal planar graph of order n with exactly four hamiltonian cycles.
2. Every $4-$connected maximal planar graph on n vertices contains at least $\dfrac{n}{\log_2 n}$ hamiltonian cycles.

A. Thomason [296] provided the answer to several interesting problems. Smith (see [310]) proved that in a cubic graph, the number of hamiltonian cycles containing a given edge is even. Thomason [296] proved that if all vertices of G, with the possible exception of two (say u and v), have odd degree, then the number of hamiltonian paths from u to v is even. Thomason also generalized in several ways the result of Kotzig (see [57]) that in a bipartite cubic graph, the total number of hamiltonian cycles is even.

Sloane [289] asked if the existence of a pair of edge-disjoint hamiltonian cycles in G implied the existence of another such pair. Thomason [296] answered this positively.

Theorem 5.12 [296]. In a $4-$regular graph of order $n \geq 3$, the number of pairs of edge-disjoint hamiltonian cycles in which two fixed edges lie in the same cycle is even.

Nincak [236] proved that if G contains k edge-disjoint hamiltonian cycles, then G contains at least $k(2k - 1)$ hamiltonian cycles. Thomason [296] showed the following.

Theorem 5.13 [296]. If a $2k-$regular graph G of order $n \geq 3$ has a decomposition into k edge-disjoint hamiltonian cycles, then

1. each edge of G is in at least $3k - 2$ hamiltonian cycles,
2. G has at least $k(3k - 2)$ hamiltonian cycles, and
3. G has at least $(3k - 2)(3k - 5) \cdots (7)(4)(1)$ hamiltonian decompositions.

Tomescu [306] considered this question for regular graphs.

Theorem 5.14 [306]. Let G be an $m-$regular graph of order $2m - k$ $(mk = 0 \ mod \ 2)$.

1. If $k \geq 1$ and $m \geq 3k$, then each edge of G is contained in at least $(m-1)(m-2)\cdots(m-k)$ hamiltonian cycles of G.
2. The graph G has at least $\frac{1}{2}(m!/(m-k-1)!)$ hamiltonian cycles.

Finally, Horák and Továrek [163] studied the number of hamiltonian cycles in complete k-partite graphs. They obtained a recursive formula for such graphs. Using this, they were able to show the following.

Theorem 5.15 [163]. Let G be a graph of order n with $\beta_0(G) \geq m$. If $H(G)$ is the number of hamiltonian cycles in G, then

$$H(G) \leq \frac{1}{2}(k-m)! \prod_{i=2}^{m} (k-m+1-i).$$

Section 6 Closure

As mentioned in Section 0, Bondy and Chvátal extended Ore's Theorem with the idea of the (degree) closure. Their insight opened the door for others to explore similar extensions. It is now natural to consider a closure operation for any adjacency result. Over the last few years, several such closures have been investigated.

Zhu and Tian [340] provided a strengthening of the degree closure with two results that guarantee the degree closure is complete.

Theorem 6.1 [340]. Let k, n be natural numbers with $k \leq 2n-4$. Let G be a simple graph with vertex set $V(G) = \{v_1, v_2, \ldots, v_n\}$. Suppose that no indices i and j satisfy the following conditions

i. $i < j$, $deg\, v_i \leq j+k-n$, $deg\, v_j \leq j+k-n-1$, $deg\, v_i + deg\, v_j \leq k-1$ and $i+j \geq 2n-k$.

ii. If $i+j \geq 2n-k+1$, then $v_i v_j \notin E(G)$. If $i+j = 2n-k$, $v_s v_t \notin E(G)$ $(1 \leq s \leq i,\ 1 < t \leq j)$.

Then $C_k(G) = K_n$.

Theorem 6.2 [340]. Let $d_1 \leq d_2 \leq \cdots \leq d_n$ be the degree sequence of a simple graph G and k $(\leq 2n-4)$ a positive integer. If for every i satisfying

$$k-n < i < \tfrac{1}{2}k,\quad d_{n-k+i} \leq i,$$

one of the following three conditions holds:

i. $d_{n-i} \geq k-i$,

ii. $d_{n-i} = k-i-1$, $d_{n-i+1} \geq k-i$, and there exists an integer r ($1 \leq r \leq n-k+i$) such that $d_{n-k+i+r} < k-i-r$ and

$$\sum_{j=1}^{r} (r+1-j)(\,|S(n-j)| + |S(k-i-j)|\,) \geq \sum_{j=1}^{r} d_{n-k+i+j} + 1,$$

iii. $d_{n-i} = d_{n-i+1} = k-i-1$, and there exists an integer r ($1 \leq r \leq n-k+i$) such that $d_{n-k+i+r} < k-i-r$, $d_{n-k+i+j} \geq i+j$ ($j = 1, 2, \ldots, r$) and

$$\sum_{j=1}^{r} (r+1-j)(\,|S(n-j)| + |S(k-i-j)|\,) \geq \sum_{j=1}^{r} d_{n-k+i+j} + r(\,i - |S^{+}(k-i)|\,) + 1,$$

where $S(m)$, $S^{+}(m)$ denote the set of all vertices with degree m and $\geq m$, respectively.

Then, $C_k(G) = K_n$.

For nonadjacent vertices $a, b \in V$, let $T_{ab} = \{ x \in V \mid a, b \not\in N(x) \}$ and let $\bar{\alpha}_{ab} = 2 + |T|$ and $\delta_{ab} = \min_{v \in T} deg\ v$. Let the *semi-independence number,* $\bar{\alpha}(G) = \min \bar{\alpha}_{ab}$, where the minimum is taken over all pairs of nonadjacent vertices *a* and *b*.

Ainouche and Christofides [4] defined the $k-$ *dual closure* of a $(k+2)-$connected graph, denoted $C_k^*(G)$ to be the smallest graph *H* of order *n* such that *G* is a spanning subgraph of *H* and for all $ab \in E$, there exists an index *i*, with $i \geq \max(1, \lambda_{ab} - k - 1)$ such that

$$\bar{\alpha}_{ab}(H) > deg_T\, x_i - k.$$

Theorem 6.3 [4] . Let a, b be two nonadjacent vertices of a $2-$connected graph G and let $T = \{ x_i \mid a, b \not\in N(x_i) \}$. If there is no index k such that $k \geq \max(1, \lambda_{ab} - 1)$, $deg_T\, x_k < \bar{\alpha}_{ab}$, then *G* is hamiltonian if, and only if, $G + ab$ is hamiltonian.

Direct consequences of this result are the following.

Theorem 6.4 [4]

1. A graph *G* is hamiltonian if its 0-dual closure is complete.
2. A graph *G* is traceable if $C_{-1}^*(G)$ is complete.
3. A graph *G* is $s-$hamiltonian if $C_s^*(G)$ is complete.
4. A graph *G* is hamiltonian if $\bar{\alpha}(G) \leq k(G)$.

They also make the following conjectures.

Conjecture [4]

1. Let a, b be two nonadjacent vertices of a $k-$connected graph G and let α_{ab} be the maximum cardinality of an independent vertex set of G containing both a and b. If $\alpha_{ab} \leq k$, then G is

hamiltonian if, and only if, $G + ab$ is hamiltonian.

2. Let a, b be two nonadjacent vertices of a $2-$connected graph G. If $deg_T\, v_i \geq i + 2$, for all $i \geq \max\,(1, \lambda_{ab} - 1)$, then G is hamiltonian if, and only if, $G + ab$ is hamiltonian.
3. Let a, b be two nonadjacent vertices of a $2-$connected graph G. If $deg\, x_i) \geq 3 + \lambda_i^T$ for all $x_i \in T$, then G is hamiltonian if, and only if, $G + ab$ is hamiltonian.

Schiermeyer [267],[268] introduced yet another, more powerful closure. The basis of his closure is the following result.

Theorem 6.5 [267] Let G be a graph of order $n \geq 5$ and a and b two nonadjacent vertices with $deg\, a + deg\, b < n$. If $G + ab$ contains a hamiltonian cycle, then G will also contain a hamiltonian cycle if one of the following conditions holds:

i. If $deg\, a + deg\, b = n - 1$ (resp. $n - 2$), then there exists exactly one (resp. zero) vertex $(\neq a$ or $b)$ which is adjacent to both a and b and such that $G[\,N(a)\,]$ is disconnected. (Here $G[\,N(a)\,]$ is the graph induced by $N(a)$.)

ii. There exists two vertices $x_1, x_2 \in V - \{\, a, b\,\}$ with $deg\, x_1 \geq 3$ or $deg\, x_2 \geq 3$, $x_1, x_2 \in N(a)$ and $x \in N(b)$ for every $x \in N(x_1) \cup N(x_2)$, $x \neq a, b$.

iii. There exists two vertices $x_1, x_2 \in V - \{\, a, b\,\}$, $ax_1, ax_2, x_1x_2 \in E$, where $bx_i \in E$, $deg\, x_i \geq 4$, $deg\, x_{3-i} \geq 3$ for $i = 1$ or $i = 2$, $xb \in E$ for every $x \in N(x_1) \cup N(x_2)$, $x \neq x_1, x_2, a$, and $\{\, x \in N(x_1) \mid x \neq x_2, a, b\,\} = \{\, y \in N(x_2) \mid y \neq x_1, a, b\,\}$.

iv. There exists a vertex $x \in V - \{\, a, b\,\}$ with the properties of x_1 or x_2 from II and with the following property: Let $s = |\{\, y \in V - \{\, a, b, x\,\} \mid deg\, y = n - 1\,\}|$ $s \geq 0$, and let $y_1, \ldots, y_s$ be these vertices. Then

   i. $G - \{\, a, y_1, \ldots, y_s\,\}$, $s \geq 1$ or

   ii. $G - \{\, a, y_1, \ldots, y_s, w\,\}$ for a vertex $w \in V - \{\, a, b, x, y_1, \ldots, y_s\,\}$ or

   iii. $G - \{\, a, y_1, \ldots, y_s, w_1, w_2\,\}$ for two vertices $w_1, w_2\ V - \{\, a, b, x, y_1, \ldots, y_s\,\}$

   is disconnected and contains exactly (i) $s + 1$ or (ii) $s + 2$ or (iii) $s + 3$ components with x and b belonging to one component.

v. There exists a vertex $x \in V - \{\, a, b\,\}$, $deg\, x \geq 3$, $ax, bx \in E$, and $G[\,N(x)\,] + ab$ is complete.

vi. There exists two vertices $x_1, x_2 \in V - \{\, a, b\,\}$ with $deg\, x_1, deg\, x_2 \geq 3$, $ax_1, bx_2 \in E$ and for every $x \in N(x_1)$ with $x \neq x_2$ and for every $y \in N(x_2)$ with $y \neq x_1$ and $x \neq y$: $xy \in E$.

vii. There exist two vertices $x_1, x_2 \in V - \{\, a, b\,\}$, $x_1, x_2 \in E$ and for every $x \in N(x_1)$, $x \neq a, b$: $ax \in E$, for every $y \in N(x_2)$, $y \neq a, b$: $by \in E$.

viii. There exist two vertices $x_1, x_2 \in V - \{ a, b \}$, $ax_1, bx_2, x_1 x_2 \in E$ and for every $x, y \in N(x_1)$, for every $x, y \in N(x_2)$: $xy \in E$, $\{ x, y \} \neq \{ a, b \}$.

ix. The graph G is $2-$connected but not $3-$connected and $k(G - \{ a, b \}) = 2$.

x. There exist two vertices $x_1, x_2 \in N(a)$ (resp $N(b)$) with $deg\ x_1 = deg\ x_2 = 2$.

Recursively joining pairs of nonadjacent vertices a and b which satisfy either $deg\ a + deg\ b \geq n$ or one of the conditions (i) - (x) of Theorem 6.5, produces the *strong closure* $C_n^{\prime}(G)$. Schiermeyer showed that if the strong closure is complete, then G is hamiltonian. He also verified that the strong closure detects all hamiltonian graphs detected by the degree closure, dual closure and several other well-known results.

Another closure was introduced very recently by Asratyan and Khachatryan [15]. They defined the $(k, r)-$*closure* ($k \geq 2$) of G to be the supergraph H of G with the property that

$$| N_H^1(u) \cap N_H^1(v) | < 1 + \sum_{j=2}^{k} | N_H^j(u) - N_H^1(v) | + r$$

for all $uv \notin E$ with $dist_H (u, v) = 2$. (Here $N_H^1(u) = \{ x \mid dist(u, v) = j \}$.)

There are several interesting features of the $(k, r)-$ closure. First, it takes into account the extended neighborhood structure. Second, the closure is not uniquely defined (as in the other cases). For example, the graph of Figure 6.1 has two $(2, 0)-$closures, namely $G + uv$ and $G + uw$.

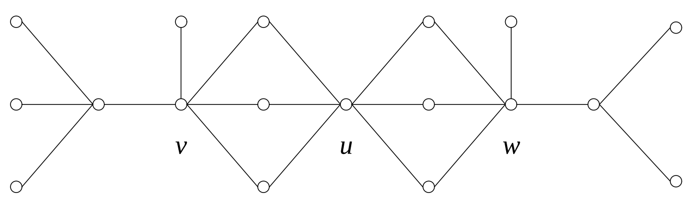


Figure 6.1 A graph with two $(2,0)-$closures.

They also verified that the property of containing a hamiltonian cycle is $(2, 0)-$stable and that the $(k, r)-$closure generalizes the degree closure as well as Fan's Theorem. It would be interesting to know its relationship to the dual and the relationship between the strong closure and the $(k, r)-$ closure. This is a problem that should be investigated.

Finally, let me mention that the neighborhood closure (based on the generalized degree condition) has also been investigated (see [106]). However, this closure is not as effective for hamiltonian properties as some of the others.

Section 7 Miscellaneous Topics

In this section I will consider several special hamiltonian problems. These will be no means exhaust such topics or even the results known on these topics. Rather, I hope merely to indicate the diversity of problems available and the many possible questions still to be asked.

In 1968, Lovász [207], conjectured that every connected vertex-transitive graph contained a hamiltonian path. This conjecture has been verified for several special orders and classes, and except for a few notable exceptions, such graphs contain a hamiltonian cycle. Babai (see [56] or [202]) proved this conjecture for graphs with prime order $p > 2$. This follows from the work of Turner [309]. Babai [20] also showed that connected, vertex-transitive graphs of order $n \geq 4$ always contain a cycle of length at least $(3n)^{1/2}$. Alspach [6] showed that every connected vertex-transitive graph of order $2p$ contained a hamiltonian cycle, unless the graph is the Petersen graph. Marusic [212] has shown that every connected vertex-transitive graph of order p^2, p^3, $2p^2$ or $3p$ have a hamiltonian cycle; while Marasic and Parsons [213] showed graphs of order $5p$ (and $4p$) have a hamiltonian path.

Babai [19] raised the problem of constructing an infinite family of connected vertex-transitive graphs that are nonhamiltonian. To date, only a few such graphs have been found. The Petersen graph, the Coxeter graph and the two graphs obtained from these by replacing each vertex by a triangle are the simplest such graphs. Thomassen (see [37]) conjectures that there are only finitely many such graphs.

Lipman [202] took a different approach. He considered graphs with a certain automorphism group, rather than a certain order. Let $Aut\ G$ denote the full automorphism group of the graph G and let Γ be a group of permutations on $V(G)$. We say Γ acts transitively if Γ has only one orbit. Using this approach Lipman was able to obtain a stronger general result.

Theorem 7.1 [202].

1. Let $\Gamma \leq Aut\ G$ be transitive on $V(G)$ and nilpotent. Then G has a hamiltonian path.
2. If G is a connected, vertex-transitive graph and $|V(G)| = p^k$, p a prime, then G has a hamiltonian path.

Another interesting class of graphs are the generalized Petersen graphs, $GP(n, k)$, for $n \geq 2$ and $1 \leq k < \frac{n}{2}$, with

$$V = \{ u_0, u_1, \ldots, u_{n-1}, v_0, v_1, \ldots, v_{n-1} \}$$

and all edges of the form $u_i u_{i+1}$, $u_i v_i$ and $v_i v_{i+k}$, for $) \leq i \leq n-1$, where all subscripts are taken modulo n.

Robertson [260] proved that $GP(n, 2)$ is hamiltonian unless $n \equiv 5 \ mod\ 6$. Castagna and Prins [66] conjectured that all $GP(n, k)$ were hamiltonian except for those isomorphic to $GP(n, 2)$ for $n \equiv 5 \ mod\ 6$. In [9], this conjecture was verified, provided n is sufficiently large. Finally, Alspach [7] succeeded in verifying the conjecture and extending the definition of $GP(n, k)$ to the nontrivalent case $n = 2k$, he showed that $GP(n, \frac{n}{2})$ is not hamiltonian if, and only if, $n \equiv 0 \ mod\ 4$ and $n \geq 8$.

Another related class of sparse regular graphs have proven to be a little more difficult to handle. The *odd graphs*, O_k, have as their vertex set the $(k - 1)$–element subsets of a $(2k - 1)$–element set (denote these subsets as $P_{k-1}(2k - 1)$). Two vertices X and Y are adjacent in O_k if $X \cap Y = \phi$. The odd graph O_3 is isomorphic to the Petersen graph.

The *Boolean graphs*, B_k, have vertex set $V = P_{k-1}(2k - 1) \cup P_k(2k - 1)$ and X is adjacent to Y in B_k if $X \subset Y$. Thus, B_k is the graph formed from the middle levels of the Boolean lattice of a $(2k - 1)$–element set by identifying the subsets as vertices with adjacency if, and only if, one set is a proper subset of another.

Several interesting problems have arisen on these two classes of graphs. We say one of these graphs has a hamiltonian decomposition if its edge set can be partitioned into hamiltonian cycles or hamiltonian cycles and a perfect matching.

Conjecture [221]: The graph O_k $(k \geq 4)$ has a hamiltonian decomposition.

Conjecture (Erdös, see [90]): The graph B_k $(k \geq 2)$ is hamiltonian.

Conjecture [95]: The graph B_k $(k \geq 2)$ has a hamiltonian decomposition.

To date, the graphs O_4, O_5 and O_6 have been shown to have a hamiltonian decomposition (see [221]), while O_7 and O_8 have been shown to be hamiltonian (see [221] and [214] respectively).

As for the Boolean graphs, B_1, B_2 and B_3 are easily seen to have a hamiltonian decomposition, while B_4 was shown to have such a decomposition in [184]. The Boolean graphs B_5, B_6, B_7 and B_8 were all shown to be hamiltonian in [95]; while independently Dejter [90] showed B_8 and B_9 were hamiltonian.

In [95], it was noted that B_k is isomorphic to $O_k \times K_2$, where $\times$ here represents the weak product, that is, (x_1, y_1) is adjacent to (x_2, y_2) in $G_1 \times G_2$ if, and only if, x_1 is adjacent to x_2 in G_1 and y_1 is adjacent to y_2 in G_2.)

Another interesting hamiltonian problem was posed by R. Roth (personal communication).

Problem : Let $B_i(2k - 1)$ be the graph obtained from symmetrically opposed levels of the Boolean Lattice of an odd ordered set. That is,

$$V(B_i(2k - 1)) = P_i(2k - 1) \cup P_{2k-1-i}(2k - 1)$$

and X is adjacent to Y if $X \subset Y$. Which generalized Boolean graphs B_i $(i \geq 1)$ are hamiltonian?

R. Roth (personal communication) conjectures that each B_i $(i \geq 1)$ is hamiltonian.

Another generalization along these lines is due to Chen and Lih [74]. They define a *uniform subset graph* $G(n, k, t)$ to have all k–subsets of an n–set as vertices and two vertices are joined by an edge if, and only if, the corresponding k–subsets intersect in exactly t elements. For special values of n, k and t, the uniform subset graphs have appeared under various names. The Johnson schemes $J(n, k)$ in the theory of association schemes is $G(n, k, k - 1)$ (see [226]). Kneser's graph (see [204]) is $G(2n + k, n, 0)$;

while $G(2k - 1, k - 1, 0)$ are the odd graphs. Chen and Lih make the following conjecture.

Conjecture [74]. The graph $G(n, k, t)$ is hamiltonian for any admissible triple (n, k, t) except $(5, 2, 0)$ and $(5, 3, 1)$.

Heinrich and Wallis [152] proved the following:

1. The graph $G(n, k, 0)$ is hamiltonian if $n \geq k + \dfrac{k2^{1/k}}{(2^{1/k} - 1)}$.
2. The graph $G(n, k, 0)$ is hamiltonian for
   a. $k = 1, n \geq 3$,
   b. $k = 2, n \geq 6$,
   c. $k = 3, n \geq 7$.

Chen and Lih [74] settle their conjecture for the cases $(n, k, k - 1)$, $(n, k, k - 2)$ and $(n, k, k - 3)$ as well as for suitably large n when k is given and t equals 0 or 1. This is not strong enough, however, to help with the odd graph conjecture.

Yet another interesting class of graphs defined from products are the *hypercubes* H_k, where $H_k = H_{k-1} \times K_2$ and where $H_1 = K_2$ (note that here $\times$ denotes the usual cartesian product). It has long been known that H_k is hamiltonian, when $k \geq 2$. A Gray code can be used to find the hamiltonian cycle. However, it was conjectured that the hypercubes actually had a hamiltonian decomposition. That this is true is a consequence of a more general result of Aubert and Schneider [17].

Theorem 7.2 [17] Let C be a cycle and let G be a graph whose edge set can be decomposed into 2 hamiltonian cycles. Then $G \times C$ (cartesian product) can be decomposed into 3 hamiltonian cycles.

Corollary 7.3 [17]

a. The graph $C_r \times C_s \times C_t$ is decomposable into 3 hamiltonian cycles.

b. The graph $K_{2s+1} \times K_{2s+1} \times K_{2s+1}$ is decomposable into $3s$ hamiltonian cycles.

c. The graph $K_{2r} \times K_{2r} \times K_{2r}$ is decomposable into $3r - 2$ hamiltonian cycles.

Hamiltonian properties of a variety of graph products have been studied in detail. In particular, Teichert (see [292,293,294,295]) has studied these properties in detail.

Other graph valued functions also can be studied. For example, powers of graphs lend themselves naturally to hamiltonian problems since the higher the power (up to the diameter), the more dense the graph becomes. Powers of graphs were studied by Paoli [245].

Given a connected graph G, if we consider the sequence of graphs

$$G, L(G), L^2(G), L^3(G), \cdots$$

where $L^i(G) = L(L^{i-1}(G))$, then for $G \neq P_k$, the graphs in this sequence eventually become hamiltonian. The minimum i such that $L^i(G)$ is hamiltonian is called the *hamiltonian index of G.* Clark and Wormald [84] suggest studying not only the hamiltonian index, but similar concepts for edge-hamiltonian and hamiltonian-connected line graphs.

Many results related to the hamiltonian index have appeared. Lai [193] has most recently studied this topic. He also considered contractions and their relation to hamiltonian line graphs in [192].

Zhan [330] provided a result on hamiltonian-connected line graphs. Other higher hamiltonian properties of line graphs were studied in [58] and [60].

Theorem 7.3 [330] If G is $4-$edge connected, then $L(G)$ is hamiltonian-connected.

Another special class that has received considerable attention recently is the following: A graph G is said to be *hamiltonian-connected from a vertex v*, if a hamiltonian path exists from v to every other vertex $w \neq v$. In [88], a recent survey of results on such graphs is given.

Another strong hamiltonian property involves the existence of cycles through specified edges or vertices. Lovász [205] conjectured that if G is $k-$connected ($k \geq 2$), $F = \{ e_1, \ldots, e_k \}$ are independent edges of G and $G - \{ e_1, \ldots, e_k \}$ is connected when k is odd, then G contains a cycle using all the edges of F. In [206], he proved this conjecture for $k = 3$. Häggkvist and Thomassen [145] proved a weakened form of this conjecture requiring the graph to be $(k+1)-$connected.

Theorem 7.4 [145]

i. If L is a set of k independent edges in G such that any two vertices incident with L are connected by $k+1$ internally disjoint paths, then G has a cycle containing all edges of L.

ii. If G is a $(\beta_0 + k)-$connected graph, then any set of k independent edges of G is contained in a cycle.

Conjecture [145]. If G is a $\beta_0(G)-$connected graph and L is a set of independent edges such that $G - L$ is connected, then G has a cycle containing all edges of L.

Häggkvist [143] also studied a related problem. We say G is *F-hamiltonian (F-semihamiltonian)* if

i. F is an set of independent paths,

ii. F is contained in a hamiltonian cycle (path).

Theorem 7.5 [143]. Let F be a $1-$factor of G.

i. If G satisfies $\sigma_2 \geq n+1$, then G is $F-$hamiltonian.

ii. If G satisfies $\sigma_2 \geq n-1$, then G is $F-$semihamiltonian.

Häggkvist [143] also studied the degree sum of pairs of edges (another generalized degree approach) in relation to F–hamiltonian graphs. The reader interested in this should also see Woodall [323]. Cycles and paths through specified vertices has also been studied. Here I shall mention only the following: Bondy and Lovász [55] proved that a $(k + 1)$–connected nonbipartite graph contains an odd cycle through any k specified vertices. Locke [203] showed that in an $(r + 2)$–connected graph G with $\delta(G) \geq d$ and $|V(G)| \geq 2d - r$, any path Q of length r and any vertex y not on Q are contained in a cycle of length at least $2d - r$. In [97] the following were shown:

Theorem 7.6 [97]. Let G be a k–connected ($k \geq 2$) graph with $\delta(G) \geq d$ and order at least $2d$. Let X be a set of k vertices of G Then G has a cycle C of length at least $2d$ such that every vertex of X is on C.

Theorem 7.7 [97]. Let G be a k–connected graph ($k \geq 3$) with $\delta(G) \geq d$ and order at least $2d - 1$. Let x and z be vertices of G and Y be a subset of $k - 1$ vertices of G. Then G has an $x - z$ path P of length at least $2d - 2$ such that every vertex of Y is on P.

Tutte [312] showed that all 4–connected planar graphs are hamiltonian. Tutte [310] also showed that some 3–connected planar graphs are nonhamiltonian. Horton (see [56]) and Ellingham and Horton [100] have constructed nonhamiltonian bipartite cubic 3–connected graphs. However, a longstanding conjecture remains.

Barnette's Conjecture [205]. Every cubic 3–connected bipartite planar graph is hamiltonian.

In [162], some results lending support to Barnette's Conjecture are discussed. In particular, all such graphs of order at most 66 are shown to be hamiltonian. They also provide further references to related work.

Let S generate the group Γ. Define the *Cayley graph* $Cay_S(\Gamma)$ as follows: the vertex set V corresponds to the elements of Γ and (x, xs) is an arc of $Cay_S(\Gamma)$ with initial vertex x and terminal vertex xs whenever $x \in \Gamma$ and $s \in S$.

Several natural problems concerning Cayley graphs have been studied.

Problems.

1. For what generating sets S does the group Γ have a hamiltonian Cayley graph?
2. Which groups Γ have the property that for all generating sets S for Γ, $Cay_S(\Gamma)$ contains a hamiltonian path?

A great deal of work has been done in this area. Witte and Gallian [319] wrote an excellent survey article on this subject. The interested reader should begin there.

Finally, let me mention one last variation. For any integer k, define a graph to be *pancyclic modulo k* if it contains a cycle of every length $mod\ k$.

Problem [87]. Characterize the $3-$connected graphs which are pancyclic modulo 3.

Theorem 7.7 [87]. If G is a planar graph and

i. $\delta(G) \geq 4$, then G is pancyclic modulo 3,

ii. $\delta(G) \geq 5$, then G is pancyclic modulo 4.

Theorem 7.8 [87]. Every $3-$connected planar graph G

i. except K_4 is pancyclic modulo 3,

ii. with $\delta(G) \geq 4$ is pancyclic modulo 4,

iii. with $\delta(G) \geq 5$ is pancyclic modulo 5.

Conjecture [87]. Every $k-$connected graph ($k \geq 3$) contains a ($0 \ mod \ k$) cycle.

Problem [87]. For a fixed k, what is the computational complexity of deciding whether or not an arbitrary graph contains an induced ($0 \ mod \ k$) cycle?

Clearly, I have only scratched the surface of many of these topics. However, a some point each survey paper must come to an end, and this seems an appropriate time for one.

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